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by Joseph Norwood, Jr.

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CALCULATION OF THE EFFECT OF VISCOUS DRAG ON THE PERFORMANCE OF A COAXIAL PLASMA GUN

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SUMMARY

It is shown that under the assumptions of ionic impact as the dominant emission process in coaxial pulsed plasma guns and the second Townsend ionization coefficient much smaller than unity, a viscous drag force results which imposes a velocity limitation on the accelerated plasma. Calculations are presented to show that for operating conditions which optimize efficiency, the effect can seriously degrade the gun performance, particularly for those cases where the stored energy is small and where the current is oscillatory during the acceleration.

INTRODUCTION

Plasma guns, particularly those with a coaxial electrode geometry, have been the subject of considerable research in the fields of thermonuclear fusion (ref. 1) and space propulsion (ref. 2). The requirement in both of these fields has been the efficient acceleration of a plasma to high velocities.

The coaxial plasma gun consists of a pair of coaxial metal electrodes separated by an insulating material at the breech end with the electrodes extending into a vacuum. A capacitor bank is connected across the electrodes through spark gap switches. A given amount of gas is injected radially through ports in the inner electrode located at the breech and assumes a distribution peaked over the ports in about 100 microseconds after the gas valve is pulsed. The capacitor bank is then switched and a discharge occurs and creates a plasma in the annular space between the electrodes. The current in the plasma interacts with the self-induced magnetic field in the gun to accelerate the plasma toward the muzzle. The effect is entirely analogous to a series electric motor, the plasma being the armature. Depending on the delay time between the pulsing of the gas valve and the switching of the capacitor bank, the gun may operate with a slug mass distribution wherein the gas is still confined to the axial region of the ports and is ionized and accelerated into a vacuum. The alternative is to give the gas a chance to diffuse down the barrel and then switch the capacitor bank. In this case, the current-carrying plasma sweeps up mass as it accelerates along the electrodes.

The early history of gun research was characterized to some extent by a cut-and-try type of approach. It later became evident, however, that the processes occurring in the plasma and possibly at the plasma-electrode boundary must be examined more fully in order to obtain sufficient understanding to be able to design guns correctly. Recent work by Bostick (ref. 3) and by Lovberg (ref. 4) typifies this new approach and has added greatly to our knowledge of the internal state of the plasma.

The plasma-electrode boundary effects have not been subjected to a thorough investigation. The causes of viscous drag suggested in reference 5 and calculated on a simple model in this paper are postulated in order to point out that electrode effects may have a very important role in the physics of plasma guns.

The specific purpose of this paper is not to explain quantitatively experimental observations but rather the influence of one effect, the viscous drag, is singled out for study.

SYMBOLS

A	cathode area, meters^2 (m^2)
B	magnetic flux density, tesla (T)
c	speed of light, meter/sec (m/sec)
C	capacitance, farads (F)
E	electric field strength, volts/meter (V/m)
e	charge on singly charged ion, coulombs (C)
F_D	drag force, newtons (N)
h	Planck's constant, joule-sec (J-sec)
I	current, ampere (amps)
k	Boltzmann's constant, joules/^{o}K (J/^{o}K)

l	distance over which plasma is accelerated, meter (m)
L	inductance, henries (H)
L_o	parasitic inductance, henries (H)
L_1	gradient of inductance, henries/meter (H/m)
M	plasma mass, kilograms (kg)
M_n	nondimensional plasma mass (defined in eq. (11))
M_o	injected propellant mass, kilograms (kg)
m_i	ion mass, kilograms (kg)
N	number of photons per second (defined in eq. (1))
n	number density per cubic meter
n_i	ion number density per cubic meter
R	ohmic resistance, ohms
r_2	outer electrode radius, meters (m)
r_1	inner electrode radius, meters (m)
t	time, seconds (sec)
T	temperature, ^oK
v	velocity, meters/sec (m/sec)
V_i	ionization potential, volts (V)
V_o	capacitor bank charging voltage, volts (V)

x axial coordinate, meters (m)

y dummy integration variable

z impact frequency, second^{-1} (1/sec)

α proportion of current carried by ions

γ Townsend secondary ionization coefficient

η efficiency

η_{max} maximum efficiency

ϕ work function, volts (V)

μ_o vacuum permeability, henries/meter (H/m)

ν frequency, hertz (H_z)

ν_o photoelectric threshold frequency, hertz (H_z)

THEORY

In a previous paper (ref. 5), it was proposed that a drag force exists which acts on an impulsively accelerated plasma in a coaxial gun in such a way as to set an upper limit to the plasma velocity which may be less than the theoretical upper limit given by the electric drift E/B. (See ref. 6.) The approximations embodied in this theory are reviewed.

The plasma is assumed to be fully ionized and the dynamical effect of collisions is ignored. The pressure of the background gas is zero; that is, the plasma is accelerated with constant mass into a vacuum. This mode of operation is known as the "slug" mode. Because of the extremely high current densities ($\approx 10^9$ amps/m^2) observed in such a device, thermionic and field emission mechanisms are discounted and it is assumed that electrons can leave the cathode only by the impact of ions or ultraviolet photons.

The Townsend secondary ionization coefficient γ is assumed to be much less than one. Somewhat scattered data on the values of γ have appeared in the literature and many of these data seem to be unreliable (ref. 7). The Townsend secondary ionization coefficient represents the number of free electrons liberated from the cathode per ion impact. This number, as seen from the data of reference 7, ranges from 10^{-3} to 10; thus, the approximation $\gamma << 1$ may be satisfied.

The ratio of the probability of the impacting ion being reflected as an ion to the probability of its capturing an electron and being reflected as a neutral is given in reference 8 as $\exp\left(\frac{\phi - V_i}{kT}\right)$ where ϕ and V_i are the work function of the material and the ionization potential of the incident particle, respectively. Since $V_i >> \phi$ for cases of interest and kT is of the order of a few electron volts only, the argument of the exponential is negative and the probability ratio is very small. This condition then implies that the ion will almost certainly be reflected as a neutral. It will be assumed that the electron is captured in the ground state and that the impacting ion does not impart sufficient energy to the lattice structure of the cathode material to cause sputtering. These assumptions are in accordance with a neglect of radiation loss processes and the requirement that the plasma mass be constant.

The neglect of radiation loss can be justified for plasmas which are not contaminated by high-Z material from electrodes or insulators. For a fully ionized hydrogen plasma at a temperature of 10 eV, the bremsstrahlung radiation from a 10-cm^3 volume is only of the order of 50 watts. This radiation is negligible compared with the power input to the gun which can be of the order of 10^8 watts.

The angle that the returning neutral makes with the cathode surface is assumed to be independent of the angle of incidence of the ion; the reflection is perpendicular on the average. (See refs. 9 to 11.) The kinetic energy corresponding to the axial component of velocity of the ion is thus lost to the cathode. The energy necessary for ionization of the neutral is assumed to be small compared with the energy available in the plasma.

These assumptions together with the Townsend model of a discharge assert that the current flowing into the anode will be due to electrons only whereas the current at the cathode will be due, except for a fraction γ, to ions. The contribution to electron production at the cathode by photoelectric effect has been ignored up to this point and must now be considered. An admittedly extreme plasma condition will be postulated in order to set an upper limit on the amount of electron current which could be produced photoelectrically. The number of photons impacting the cathode with energy above the photoelectric threshold as a function of plasma temperature is found by assuming that the plasma is radiating as a black body, a condition which may be fulfilled for a brief time in some devices (ref. 12). This number is

$$N = A \frac{2\pi}{c^2} \int_{\nu_0}^{\infty} \frac{\nu^2}{e^{h\nu/kT} - 1} d\nu \tag{1}$$

where ν_0 corresponds to the photoelectric threshold, $h\nu_0 = \phi$. It must be emphasized that this condition could only be obtained in high density ($n > 10^{17}$ cm^{-3}) plasmas consisting almost entirely of materials with a high nuclear charge. Such a condition has been observed, however, in a coaxial plasma gun. (See ref. 13.) Since for plasma temperatures on the order of 5 eV most of the impacting photons are very energetic, the efficiency of electron emission is taken as unity. Evaluating the integral by means of the tables of Debye and Zeta functions in reference 14 and taking typical values for the cathode area $A = 1$ cm^2 and the plasma temperature $kT = 5$ ev yields for the possible current at the cathode due to photoelectric electrons

$$I = \frac{2\pi e}{c^2}\left(\frac{kT}{h}\right)^3 A \int_{\phi/kT}^{\infty} \frac{y^2}{e^y - 1}\, dy \approx 10^7 \text{ amps} \tag{2}$$

In other words, the proportion of current carried by the ions at the cathode may be very small for guns which feature a large radiation flux.

By using the previous assumptions, the viscous drag caused by ion impact on the cathode can now be written as

$$F_D = z m_i v_x \tag{3}$$

where z is the number of ion impacts per second

$$z = \frac{\alpha|I|}{e} \tag{4}$$

v_x is the axial component of the average ion velocity, and α is the proportion of the current carried by the ions. The parameter α might assume any value between 0 and 1. For $\gamma << 1$ and a low radiation flux, α would approach unity, whereas either a large radiation flux or a large value of γ would cause the current to consist mostly of electrons. For this case α would be very small. The absolute value sign on the current in equation (4) is necessary since I is generally oscillatory.

Another means by which particles may reach the electrodes and produce a drag force is by thermal diffusion. In this case the impact frequency z will be given by the product of efflux velocity $\sqrt{kT/2\pi m_i}$, ion density n_i, and electrode area A. At the high ion densities encountered in coaxial guns ($n_i \approx 10^{16}$ to 10^{17} cm^{-3}), the mean free path for ions (or neutrals) is less than 1 millimeter; hence, for temperature gradients which may be typical in coaxial guns, the impacting particles will have thermal energies corresponding to only a few tenths of an eV. Thus, most of the impacting particles may be neutrals for which the assumption of diffuse reflection at the electrode surface is not likely to hold. If $A = 30\ \text{cm}^2$, $n_i = 10^{16}\ \text{cm}^{-3}$, and $kT = 0.5$ eV, the calculation for hydrogen yields $z \approx 10^{23}\ \text{sec}^{-1}$ which must be reduced because specular reflection of neutrals will yield no momentum transfer to the electrode. The impact frequency due to ion current I/e for a current of 10^5 amps will be $3 \times 10^{24}\ \text{sec}^{-1}$; hence, the effect of thermal diffusion is ignored.

The energy equation for the coaxial gun powered by a capacitor bank is

$$\frac{1}{2} CV_o^2 = \frac{\left(CV_o - \int_0^t I\,dt\right)^2}{2C} + \frac{1}{2}(L_o + L_1 x)I^2 + \int_0^t RI^2 dt + \frac{1}{2} M\left(\frac{dx}{dt}\right)^2 + \alpha \frac{m_i}{e} \int_0^x |I| \frac{dx}{dt}\, dx \tag{5}$$

where x is the coordinate along the axis of the gun, C is the capacitance of the bank, I is the current, R is the total series resistance (assumed constant), V_o is the voltage to which the capacitor bank is charged, M is the constant mass of the accelerated plasma, L_o is the parasitic inductance, and L_1 is the gradient of inductance along x,

$$L_1 = \frac{\partial L}{\partial x} \approx \frac{\mu_o}{2\pi} \log_e \frac{r_2}{r_1} \tag{6}$$

In equation (6) r_2 and r_1 are the outer and inner radii, respectively, of the electrodes of the coaxial gun and μ_o is the vacuum permeability. The parameter L_1 is approximately constant. Experiments have suggested that the plasma thermal energy is small compared with its directed energy; thus, a heating term is not included in the energy equation (5).

The momentum equation including the viscous force is

$$M \frac{d^2x}{dt^2} + \alpha |I| \frac{m_i}{e} \frac{dx}{dt} = \frac{1}{2} L_1 I^2 \tag{7}$$

Taking the derivative with respect to time of equation (5)

$$V_o I = I \frac{\int_0^t I\,dt}{C} + (L_o + L_1 x) I \frac{dI}{dt} + \frac{1}{2} L_1 I^2 \frac{dx}{dt} + RI^2 + M \frac{d^2x}{dt^2}\frac{dx}{dt} + \alpha |I| \frac{m_i}{e}\left(\frac{dx}{dt}\right)^2$$

and multiplying equation (7) by $\frac{dx}{dt}$ yields

$$V_oI = \frac{I}{C}\int_0^t I\,dt + (L_o + L_1x)I\frac{dI}{dt} + L_1I^2\frac{dx}{dt} + RI^2$$

Dividing by I and differentiating with respect to time yields

$$0 = (L_o + L_1x)\frac{d^2I}{dt^2} + 2L_1\frac{dx}{dt}\frac{dI}{dt} + L_1I\frac{d^2x}{dt^2} + R\frac{dI}{dt} + \frac{I}{C} \tag{8}$$

CALCULATION OF GUN PERFORMANCE

Equations (7) and (8) form the set which is used to describe the electrical circuit and dynamics of the system. In order to ascertain the dependence of the performance of a coaxial plasma gun on the viscous drag effect, equations (7) and (8) have been programed for a digital computer. The calculations have been made for $\alpha = 0$ and 1/2. The value 1/2 is chosen as being a representative value in view of the fact that the assumptions necessary for $\alpha = 1$ are not likely to be realized completely in a practical device. The initial conditions are

$$\left.\begin{array}{ll} I(0) = 0 & x(0) = 0 \\ \frac{dI}{dt}(0) = \frac{V_o}{L_o} & \frac{dx}{dt}(0) = 0 \end{array}\right\} \tag{9}$$

The parasitic inductance was chosen as typical of efficient design for the various capacitor banks. The gradient of gun inductance being logarithmic is relatively insensitive, and thus r_2/r_1 was chosen as $\log_e r_2/r_1 = 1$ for convenience. The ohmic resistance R was also chosen as typical of such systems, 10^{-3} ohm. The ion mass m_i is taken as that of atomic hydrogen.

The results of the calculation are shown in figures 1 to 6 and are given in table I in terms of gun efficiency, in percent, defined as

$$\eta = \frac{\frac{1}{2}M\left(\frac{dx}{dt}\right)^2}{\frac{1}{2}CV_o^2} \times 100 \tag{10}$$

where dx/dt is measured at the time the plasma has achieved a distance from the origin of 40 cm. This distance is taken as an average acceleration length for such a device. In order to minimize ohmic losses, one may wish to terminate the acceleration at the time the current oscillates through zero. This length trimming also serves to prevent the growth of current plumes from the gun and consequent transverse acceleration. Since the present study encompasses such a wide range of parameters, this method of optimum length selection is impractical in most cases and so an average practical length has been selected.

As has been noted by other authors, the problem of obtaining efficient operation in a plasma gun is essentially one of matching the inertia of the plasma to the "inertia" of the electrical circuitry. (See ref. 2.) Too low a mass of plasma will result in the plasma being ejected from the electrode region before an appreciable part of the energy of the capacitor bank has been applied. Conversely, too high a plasma mass will result in a low velocity for a given available energy; thus, $\frac{\partial L}{\partial t} = L_1 \frac{dx}{dt}$ will be small. The back electromotive force $\frac{1}{2} I \frac{\partial L}{\partial t}$ is small compared with the applied voltage; thus, the efficiency is small.

DISCUSSION OF RESULTS

In figures 1 to 6 the efficiency η has been plotted as a function of the accelerated mass. It is seen that the drag effect maximizes for low mass loading as one would expect since the velocities are greater for smaller masses than for larger.

The hump which appears on several of the curves occurs at the point where the current becomes oscillatory with increasing mass because of increasing current damping by the work performed in accelerating the plasma and by the ohmic losses. As the current becomes oscillatory, the time required to accelerate the plasma increases rapidly. The consequent drop in average acceleration tends to make the ratio of the joule heating term to the work term, in the energy equation, larger than it would otherwise be.

The optimum mass loading from the point of view of maximum efficiency is seen to be from 1 μg to 10 μg for all cases except that shown in figure 5. This value agrees within a factor of about 10 with the results of an analytical and experimental study which are shown in figure 7 replotted from the data of reference 15. In figures 6, 9, and 10 of reference 15, calorimetrically measured efficiency is plotted against a dimensionless parameter M_n which is related to M, the mass loading by

$$M_n = \frac{2l}{L_1 C^2 V_o^2} M \tag{11}$$

where l is the length over which the plasma is accelerated. For curves 1 to 6, $L_1 = 2.18 \times 10^{-7}$ H/m; for curves 7 to 9, $L_1 = 5.4 \times 10^{-7}$ H/m. For curves 1 to 3,

l = 30 cm; for curves 4 to 6, l = 46 cm; for curves 7 to 9, l = 20 cm. Curves 1, 4, and 7 correspond to $V_o = 2.5 \times 10^4$ V, whereas curves 2, 5, and 8 and curves 3, 6, and 9 have $V_o = 2 \times 10^4$ V and $V_o = 1.5 \times 10^4$ V, respectively.

A comparison of gun performance with and without viscous drag is summarized in table II. The comparison is made at the point of maximum efficiency of the curves for $\alpha = 1/2$. As can be seen from the fourth column of the table, the effect may vary from approximately 0 percent to almost 30 percent. Again the case of figure 5 is singular with an effect of less than 1 percent. It would seem that for high energy systems the circuit is less sensitive to the dynamics of the plasma and at the higher mass loadings necessary to utilize the larger energy, the $M \frac{d^2x}{dt^2}$ term dominates over the viscous drag term in equation (7).

The large drag effect for all mass in figure 2 may be explained. The drag force depends on the square root of the stored energy through the current; thus, the ratio of energy input to viscous drag to stored energy increases with decreasing energy storage and figure 2 is a case for which the stored energy is small.

CONCLUDING REMARKS

It has been shown that if ionic impact is the dominant mechanism for electron emission and the Townsend secondary ionization coefficient is much less than 1, then a drag force exists in coaxial plasma guns which may pose a limitation on the attainable plasma velocities. This effect is not limited to coaxial devices; it will also occur in a rail-type plasma gun since the difference is merely one of geometry. It may be expected to be stronger in those cases where the radiation flux is small. If an efficient circuit design is assumed, that is, a low ratio of parasitic inductance to load inductance and a high ratio of impedance to resistance, the study also concludes that peak efficiencies may be realized with plasma masses of 1 μg to 10 μg. Even at these masses, the viscous drag effect may be far from negligible, especially for moderately small stored energy and/or those cases where the transition to oscillatory current occurs at low mass numbers.

It must be emphasized that these results apply only to the "slug" mode of operation and not to a "snowplow" mode in which the accelerated mass is a function of distance. The effect of energy losses due to heating and line radiation has been ignored in this simplified treatment even though the radiation loss may be very severe.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., June 27, 1966.

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TABLE I.- COMPUTED RESULTS

Case	M, μg	$\left(\frac{dx}{dt}\right)_{\alpha=0}$, cm/$\mu$sec	$\eta_{\alpha=0}$, percent	$\left(\frac{dx}{dt}\right)_{\alpha=1/2}$, cm/$\mu$sec	$\eta_{\alpha=1/2}$, percent
Figure 1; $\frac{1}{2}CV_o^2 = 50$ joules					
1	10^{-4}	930	8.65	33.7	0.0114
2	5×10^{-4}	600	18	33.5	.0561
3	10^{-3}	490	24	34.1	.116
4	2×10^{-3}	409	33.5	32	.205
5	4×10^{-3}	320	41	30	.36
6	6×10^{-3}	280	47.1	27.7	.462
7	8×10^{-3}	251	50.3	23.3	.433
8	10^{-2}	230	52.6	24.7	.61
9	2×10^{-2}	173	59.9	33.2	2.2
10	4×10^{-2}	118	55.7	40	6.4
11	6×10^{-2}	89	47.6	40.1	9.66
12	8×10^{-2}	72.1	41.6	39.7	12.6
13	10^{-1}	64	40.9	38.6	14.9
14	5×10^{-1}	29.7	44.2	23.1	26.7
15	1	20.6	42.4	17.2	29.4
16	10	6.21	38.6	5.95	35.4
17	10^{2}	1.76	31	1.73	30
18	10^{3}	.39	15.2	.388	15.1
19	10^{4}	.049	2.4	.0489	2.4
Figure 2; $\frac{1}{2}CV_o^2 = 0.5$ joule					
20	10^{-4}	236	55.7	3.32	0.011
21	2×10^{-4}	173	59.8	1.03	.0213
22	4×10^{-4}	118	55.7	3.5	.0491
23	6×10^{-4}	89	47.5	2.2	.0291
24	8×10^{-4}	72.1	41.6	2.5	.05
25	10^{-3}	62.3	38.8	3.09	.091
26	5×10^{-3}	29.7	44.2	3.23	.522
27	10^{-2}	20.6	42.4	3.36	1.13
28	10^{-1}	6.19	38.2	3.56	12.6
29	1	1.76	31	1.5	22.6
30	10	.387	15.1	.367	13.6
31	10^{2}	.0488	2.4	.0484	2.35

TABLE I.- COMPUTED RESULTS – Continued

Case	M, μg	$\left(\frac{dx}{dt}\right)_{\alpha=0}$, cm/$\mu$sec	$\eta_{\alpha=0}$, percent	$\left(\frac{dx}{dt}\right)_{\alpha=1/2}$, cm/$\mu$sec	$\eta_{\alpha=1/2}$, percent
Figure 3; $\frac{1}{2}CV_o^2$ = 500 joules					
32	10^{-3}	570	3.25	90.5	0.082
33	10^{-2}	319	10.2	90.8	.826
34	10^{-1}	171	29.3	102	10.4
35	5×10^{-1}	104	54.2	84	35.3
36	1	82	67.2	69.5	48.3
37	5	39.3	77.2	36.5	66.6
38	10	25.8	66.5	24.7	60.8
39	10^2	7.74	60	7.63	58.3
40	10^3	2.09	43.7	2.08	43.3
41	10^4	.411	16.9	.410	16.8
Figure 4; $\frac{1}{2}CV_o^2$ = 125 joules					
42	10^{-1}	113	51.1	65.2	17.1
43	5×10^{-1}	62.2	77.4	49.1	48.2
44	1	44.4	48.9	37.6	56.7
45	5	18.4	67.7	17.2	59.2
46	10	12.3	60.5	11.9	56.6
47	10^2	3.59	51.6	3.53	49.8
48	5×10^2	1.35	36.4	1.34	36.0
49	10^3	.836	28	.832	27.8
50	5×10^3	.225	10.1	.224	10
51	10^4	.118	5.5	.118	5.5

TABLE I.- COMPUTED RESULTS – Concluded

Case	M, μg	$\left(\frac{dx}{dt}\right)_{\alpha=0}$, cm/$\mu$sec	$\eta_{\alpha=0}$, percent	$\left(\frac{dx}{dt}\right)_{\alpha=1/2}$, cm/$\mu$sec	$\eta_{\alpha=1/2}$, percent
Figure 5; $\frac{1}{2} CV_o^2 = 5000$ joules					
52	1	99	9.8	85.7	7.36
53	10	53.3	28.4	50.7	25.6
54	10^2	25.6	65.5	25.2	63.6
55	10^3	8.42	70.8	8.40	70.5
56	10^4	2.2	48.4	2.2	48.4
Figure 6; $\frac{1}{2} CV_o^2 = 50$ joules					
57	10^{-2}	99	91.8	28.3	0.8
58	10^{-1}	53.4	28.5	31.4	9.86
59	1	25.6	65.6	21.9	48
60	10	8.43	71.1	8.10	65.7
61	10^2	2.2	48.4	2.17	47.2
62	10^3	.413	17.1	.412	17
63	10^4	.0488	2.4	.0488	2.4

TABLE II.- RESULTS AT MAXIMUM EFFICIENCY

Figure	$\eta_{\alpha=0}$, percent	$(\eta_{max})_{\alpha=1/2}$, percent	$\Delta\eta$, percent	$\frac{\Delta\eta}{\eta_{\alpha=0}}$, percent	M at $\eta_{\alpha=0}$ and $(\eta_{max})_{\alpha=1/2}$, μg
1	38.6	35.4	3.2	8.28	10
2	31	22.6	8.4	27.1	1
3	77.2	66.6	10.6	13.7	5
4	67.7	59.2	8.5	12.6	5
5	70.8	70.5	.3	.42	10^3
6	71.1	65.7	5.4	7.59	10

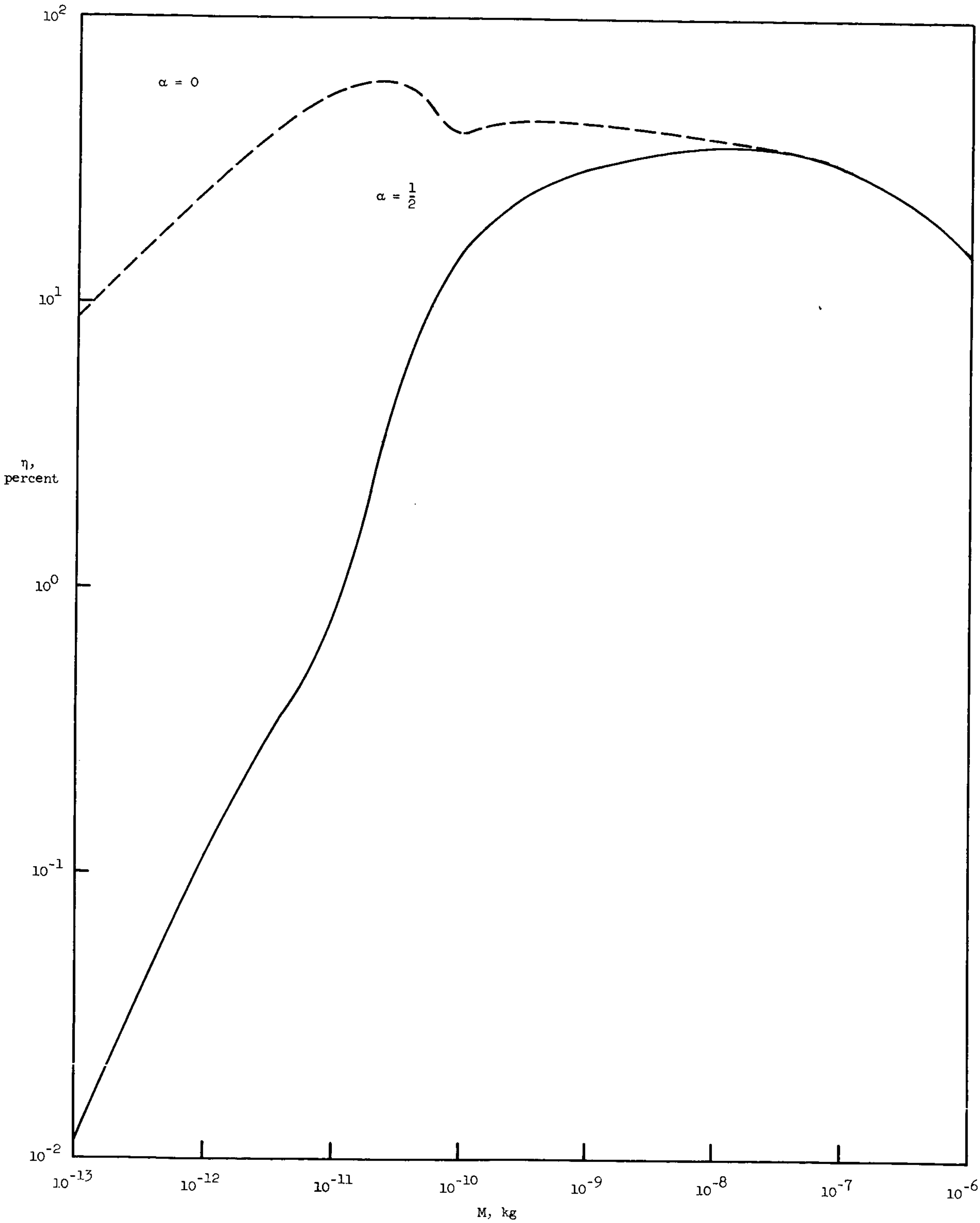

Figure 1.- Gun efficiency as a function of accelerated mass for cases 1 to 19. $C = 1\ \mu F$; $V_0 = 10$ kV; and $L_0 = 40$ nH.

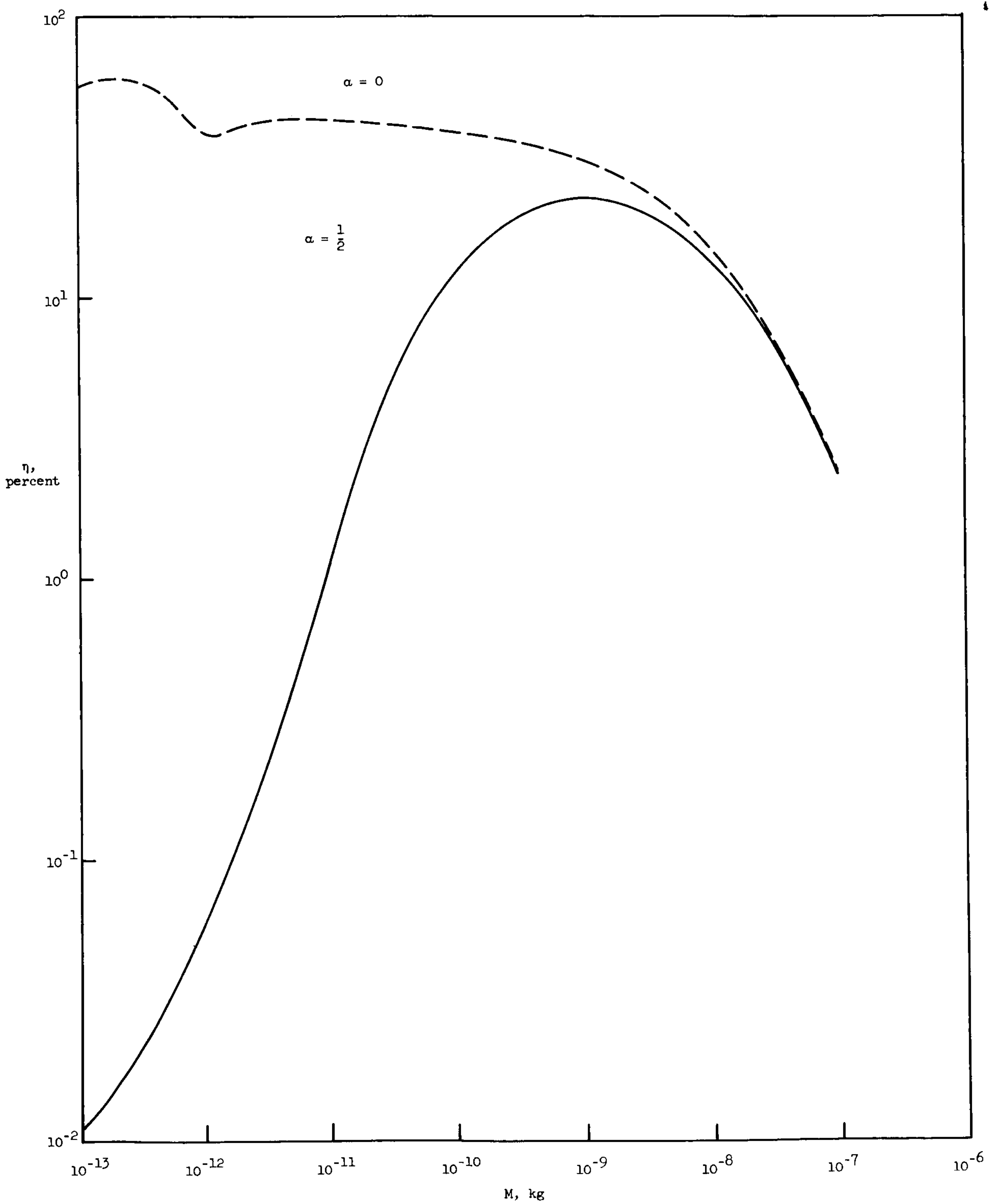

Figure 2.- Gun efficiency as a function of accelerated mass for cases 20 to 31. $C = 1\ \mu F$; $V_0 = 1$ kV; and $L_0 = 40$ nH.

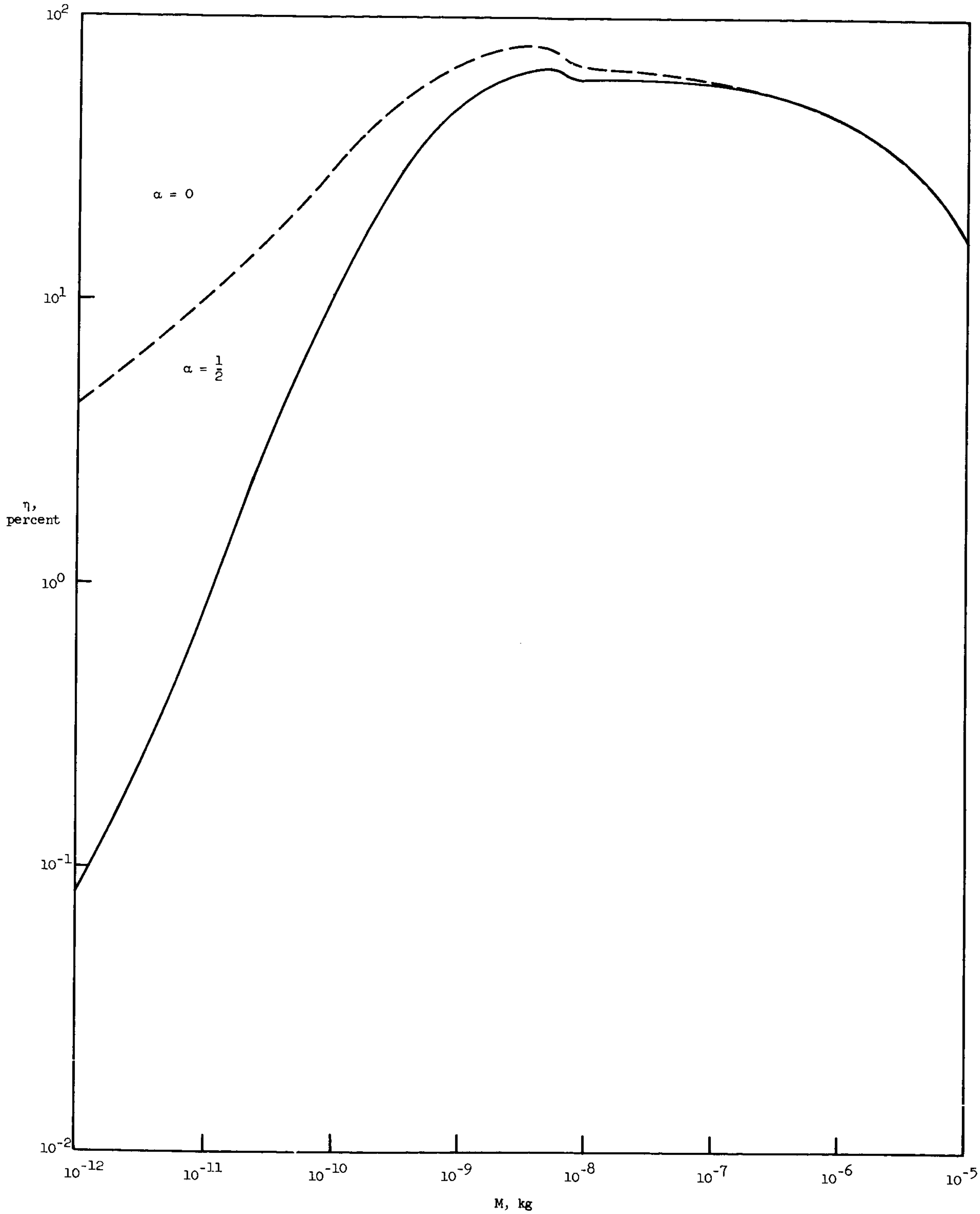

Figure 3.- Gun efficiency as a function of accelerated mass for cases 32 to 41. $C = 10\ \mu F$; $V_0 = 10$ kV; and $L_0 = 10$ nH.

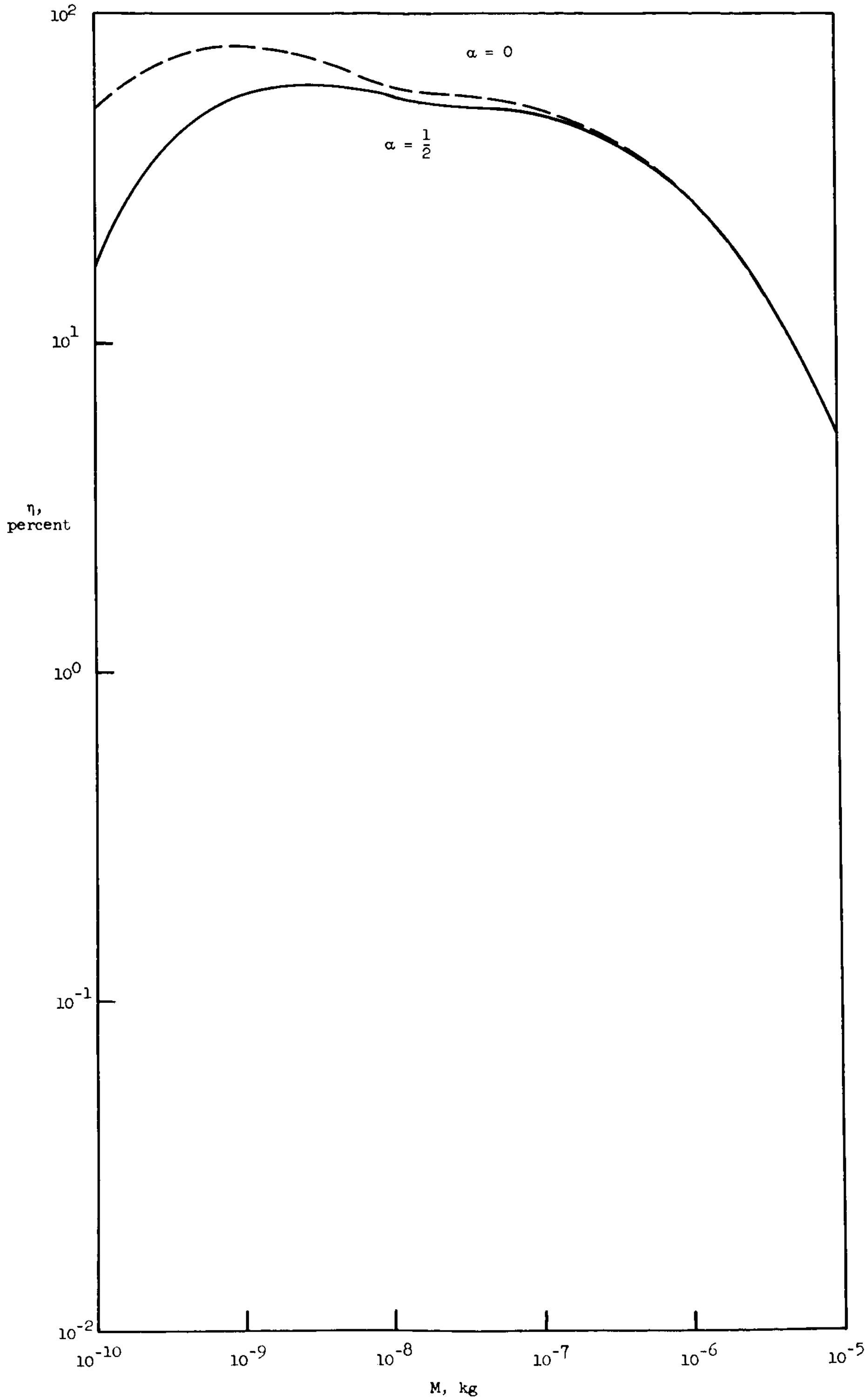

Figure 4.- Gun efficiency as a function of accelerated mass for cases 42 to 51. $C = 10\ \mu F$; $V_0 = 5$ kV; and $L_0 = 10$ nH.

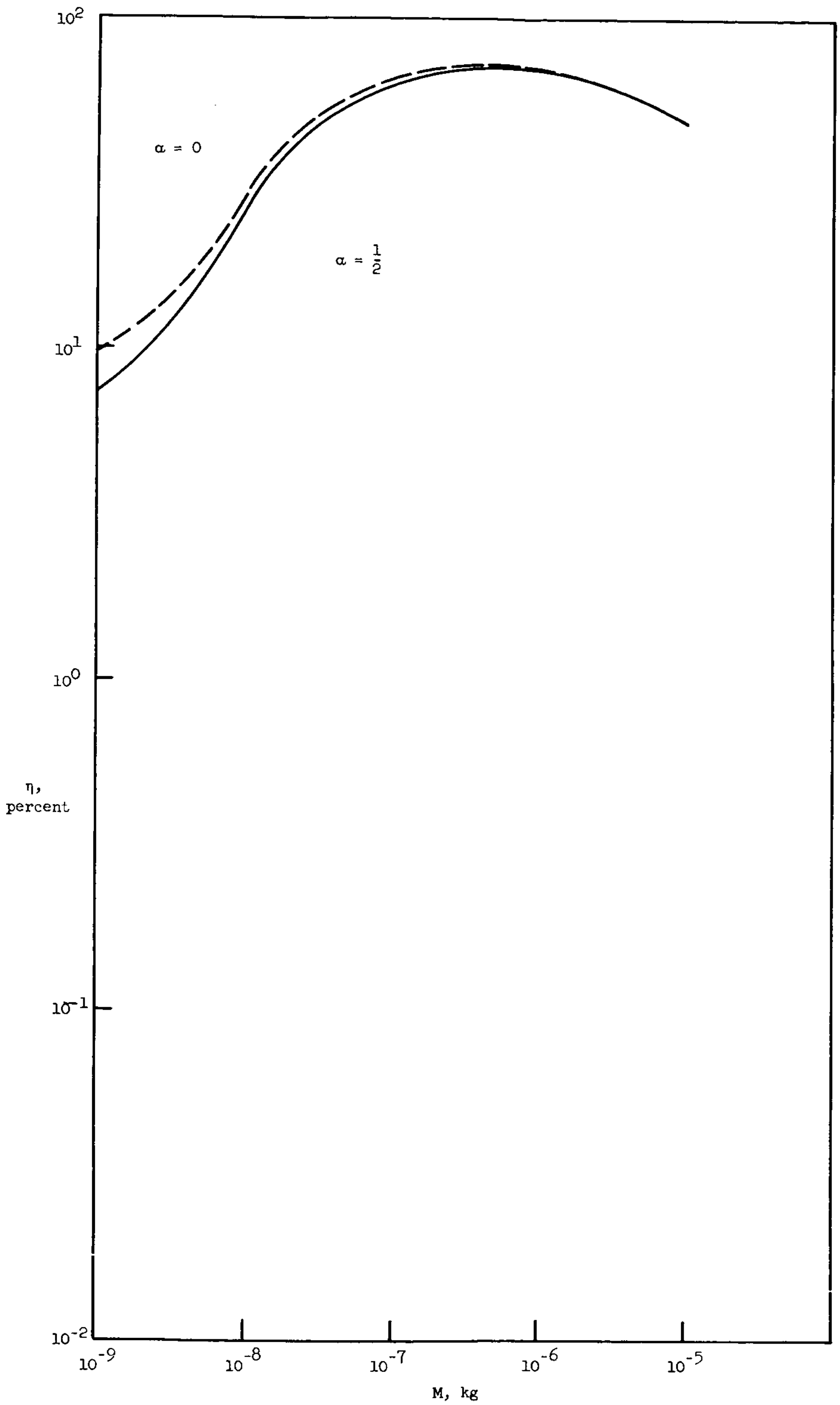

Figure 5.- Gun efficiency as a function of accelerated mass for cases 52 to 56. $C = 100\ \mu F$; $V_0 = 10$ kV; and $L_0 = 4$ nH.

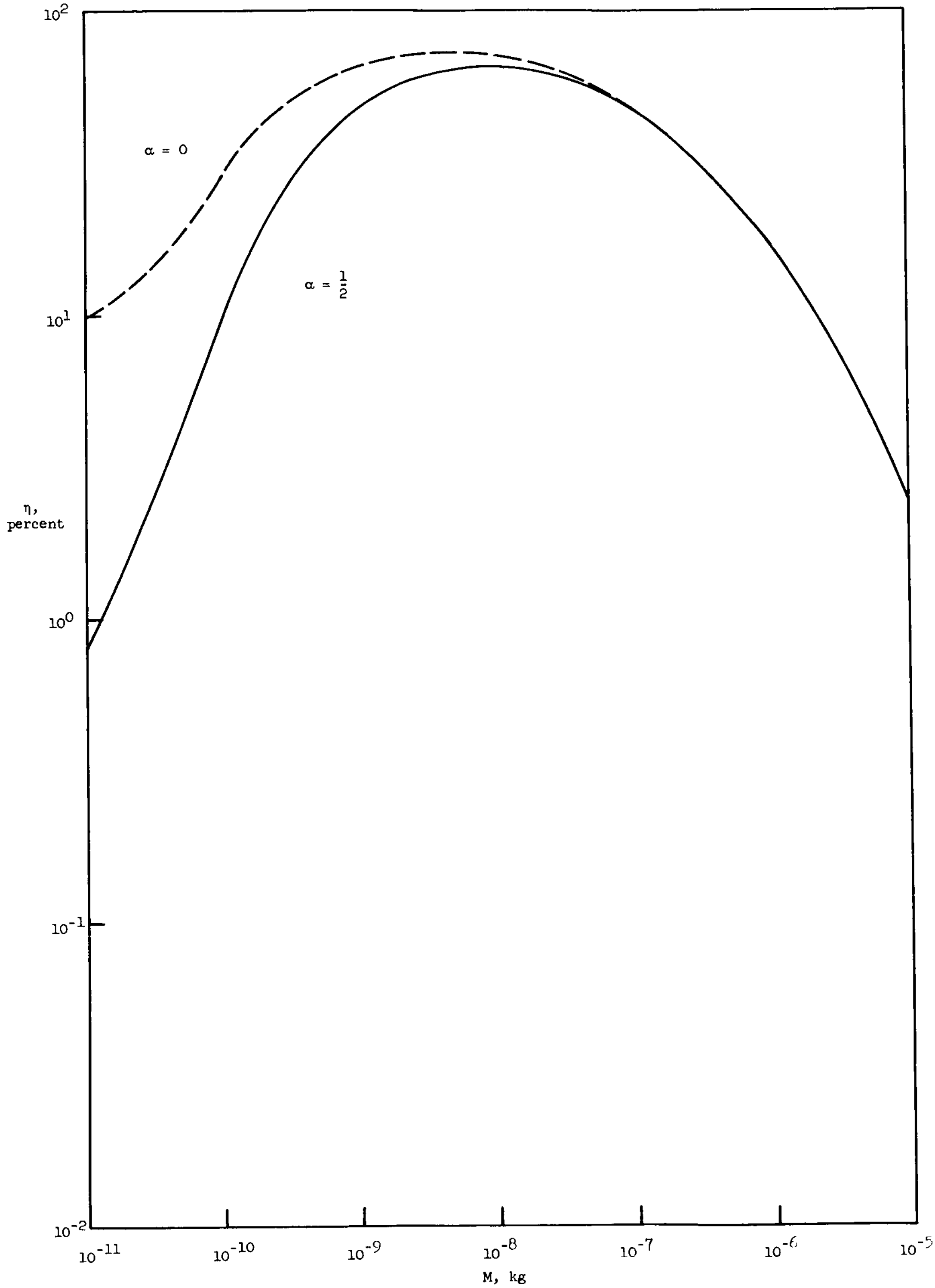

Figure 6.- Gun efficiency as a function of accelerated mass for cases 57 to 63. $C = 100\ \mu F$; $V_0 = 1$ kV; and $L_0 = 4$ nH.

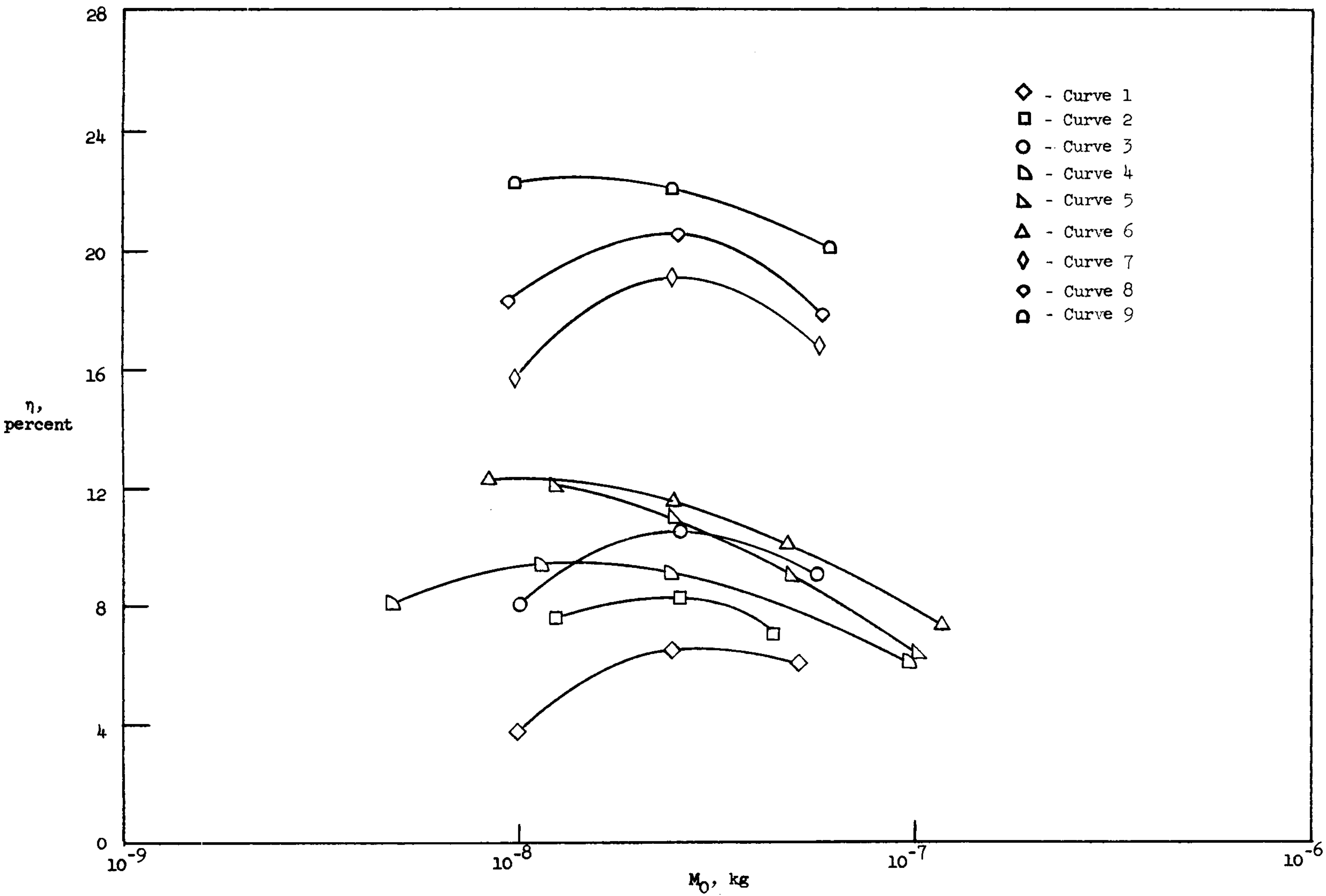

Figure 7.- Experimental efficiencies as a function of injected propellant mass computed from data of reference 15. Symbols refer to various sets of parameters; these parameters are given in the text.

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